Journal of Experimental Child Psychology 153 (2017) 168–172

Contents lists available at ScienceDirect

Journal of Experimental Child Psychology

journal homepage: www.elsevier.com/locate/jecp

Reply

Better together: Multiple lines of evidence for a link between approximate and exact number representations: A reply to Merkley, Matejko, and Ansari

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ARTICLE INFO

Article history:
Available online 20 September 2016

Keywords:
Approximate number system
Symbolic math
Causality

ABSTRACT

The results of our recent experiments suggest that temporarily modulating children's approximate number system (ANS) precision leads to a domain-specific change in their symbolic math performance (*Journal of Experimental Child Psychology*, 2016, Vol. 147, pp. 82–99). We interpreted these results as evidence for a causal relationship between ANS precision and symbolic math. In a commentary on our work, Merkley, Matejko, and Ansari argue that our methodology limits the interpretation of our results, primarily because our experiments did not meet the criteria for an intervention study as set out by What Works Clearinghouse and others. Here, we clarify the goals and limitations of our study and emphasize the variety of approaches to demonstrating causality. We argue that our goal was not to design and test an intervention or to compare the effectiveness of different treatments. Instead, we aimed to experimentally manipulate one variable (i.e., ANS acuity) and, in a randomized sample of children, observe whether this manipulation had any statistically significant effect on a dependent variable (i.e., performance on a set of symbolic math questions). We provide further analyses to support our assertion that a temporary manipulation of ANS performance does lead to a change in math performance. These results point to a causal relationship between ANS precision and math, and they suggest that further investigation of this relationship will be fruitful.

DOI of original article: http://dx.doi.org/10.1016/j.jecp.2016.07.008

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http://dx.doi.org/10.1016/j.jecp.2016.09.005
0022-0965/

How do our universal intuitions about number and quantity contribute to formal mathematics if they contribute at all? A variety of work—beginning with that of Halberda, Mazzocco, and Feigenson (2008)—has shown that children's and adults' intuitive number sense, supported by their approximate number system (ANS), correlates with performance on standardized and non-standardized math assessments even when controlling for other cognitive abilities (for reviews, see Chen & Li, 2014; Feigenson, Libertus, & Halberda, 2013; Schneider et al., 2016). Although these results have not been without criticism (e.g., Price, Palmer, Battista, & Ansari, 2012; Sasanguie, De Smedt, Defever, & Reynvoet, 2012), there is an increasing agreement that *some* aspect of our basic machinery for representing quantity contributes to *some* aspect of formal math (Chen & Li, 2014; Schneider et al., 2016).

A variety of explanations of this correlation have been offered. Some researchers suggest that the ANS is foundational for the earliest learning about math (Libertus, Feigenson, & Halberda, 2013a; Starr, Libertus, & Brannon, 2013). Others propose that math training sharpens the ANS (Lindskog, Winman, & Juslin, 2014; but see Sullivan, Frank, & Barner, in press). And still others claim that the correlation is explained by domain-general magnitude processing (Bonny & Lourenco, 2013) or executive function abilities (Fuhs & McNeil, 2013; Gilmore et al., 2013). This range of opinions and results highlights the lively interest in this topic, with each proposal raising interesting questions and spurring productive research.

In our recent work (Wang, Odic, Halberda, & Feigenson, 2016), we explored the possibility of a causal link between the ANS and symbolic math by asking whether an experimental manipulation of the ANS would change children's performance on a set of math questions. Our ANS manipulation (based on that of Odic, Hock, & Halberda, 2014) involved asking children to complete a series of nonverbal numerical discriminations (rapidly deciding whether there were more blue or yellow dots in an array) in one of three trial order conditions. For the Easy-First condition, the task started with the easiest numerical discriminations and gradually progressed to harder ones. For the Hard-First condition, it started with quite difficult numerical discriminations and gradually progressed to easier ones. And for the Random Order condition, the task presented numerical discriminations in a randomly intermixed order of difficulty. Children were randomly assigned to one of these three conditions, with all children completing the identical trials, just in different orders. Subsequently, all children were tested with a set of symbolic math problems that was shown by previous work to correlate with ANS precision (Libertus, Feigenson, & Halberda, 2013b). Our critical finding was a main effect of condition; children in the Easy-First ANS manipulation condition performed better on the math problems than children in the Hard-First ANS manipulation condition, whereas children in the Random Order condition showed intermediate performance. A separate group of children completed either the Easy-First or Hard-First ANS manipulation and then were tested on a vocabulary task rather than a math task. We found no effect of the ANS manipulation on children's vocabulary performance. Based on these findings, we concluded that experimentally manipulating ANS precision can change math performance in a way that is specific to math rather than affecting all cognitive abilities.

These findings are, of course, just a start; there are many more things that one would like to know about how modulating ANS precision affects subsequent math performance. For example, how long do the effects of this manipulation persist? How do the size and duration of its effects compare with those of other training methods such as those of Hyde, Khanum, and Spelke (2014) and Park and Brannon (2013, 2014)? Do improvements in ANS precision also affect performance on other types of math problems? Do improvements in ANS precision affect children's math anxiety, and if so is this predictive of improvements in math performance?

Each of these questions is worth asking and answering, although no single study, or single article, will be able to address them all. We believe that the diversity of these questions, and the range of their possible answers, reveals a healthy interest and debate in the field, and we look forward to continued work from our lab and others that aims to further our understanding of the origins of mathematical thinking.

However, Merkley, Matejko, and Ansari (2016) challenge our assertions; their central claim is that our findings merely add to the long list of existing correlational studies on the approximate number system and symbolic math performance. Among other issues, Merkley and colleagues propose that the lack of baseline measures in our study leaves open the possibility that the children in our Easy-First, Hard-First, and Random Order conditions differed in ANS precision even prior to our order manipulation (i.e., that we did not successfully manipulate anything). They also suggest that between-participant designs cannot convincingly show specific transfer effects (i.e., we did not demonstrate, within participants, that the benefit children experienced during the ANS discrimination task transferred to math performance but not to vocabulary performance). Implicit in their commentary is the concern that our study reports a statistical artifact and that randomized control trials (RCTs), including those adhering to the education intervention guidelines published by What Works Clearinghouse (U.S. Department of Education, 2013), are the only legitimate way to demonstrate causality.

We agree with Merkley and colleagues (2016) that large, within-participant, multi-treatment RCTs that include pre- and post-training measures of abilities using extensive standardized assessments (such as that by Sella, Tressoldi, Lucangeli, & Zorzi, 2016) are a powerful way to obtain evidence for a causal link between a specific experience and subsequent performance. However, we strongly disagree that RCTs are the only way to assess causality. Our approach of random assignment and between-participants comparison of performance after a manipulation is a common one and a valuable one in the cognitive sciences. Because we randomly assigned children to one of three experimental conditions, it would be highly unlikely for children's prior ANS abilities to just happen to align with our assignment of condition (i.e., for children with the poorest ANS precision to just happen to be assigned to the Hard-First condition, children with intermediate ANS precision to just happen to be assigned to the Random Order condition, and children with the best ANS precision to just happen to be assigned to the Easy-First condition). Furthermore, our finding that the ordering of ANS discrimination trials significantly affected ANS precision replicates the findings of previous work (Odic et al., 2014), providing convergent evidence that the ANS manipulation changed children's performance.

Of course, our central question was whether this change in ANS performance in turn affected math performance. One of the alternatives to RCTs for building a deeper understanding of causality is to ask how a treatment can change the relationships that operate across psychological factors. For instance, a treatment that enhances ANS precision should function to reduce variance in ANS precision (e.g., making each participant better) and, thereby, may result in higher correlations between ANS precision and subsequent math performance (as a result of the variance reduction and transfer effects). By this reasoning, we predicted a stronger correlation between ANS precision and symbolic math performance in the Easy-First condition compared with the Hard-First condition. Our sample sizes are too small to show this difference convincingly, and we did not plan this analysis for our original article. But we present it here as an exploratory query that may offer converging evidence for the impact of ANS confidence hysteresis on symbolic math performance. Consistent with our prediction, we observed a positive correlation between ANS precision and math performance in children in the Easy-First condition ($r = .55$) but not those in the Hard-First condition ($r = -.02$). These correlations were not significantly different from each other due to sample size limitations, but the logic illustrates another approach to addressing causality. Although we value randomized control trials for testing proposed interventions that seek to make a lasting impact on improving cognitive abilities, we urge that a diversity of approaches is valuable for exploring (and identifying) directional effects.

Beyond these more general criticisms, Merkley and colleagues (2016) raised a number of specific concerns. For example, they suggested that our "confidence hysteresis" manipulation might not be specific to the ANS but instead may affect domain-general motivational or affective states (i.e., in line with the folk, nontechnical use of the word "confidence"; for details, see Pouget, Drugowitsch, & Kepecs, 2016). We too were concerned about this possibility; this is why we asked whether the identical confidence hysteresis manipulation affected children's vocabulary performance. It did show that there are some limits to what ANS confidence hysteresis can affect. In fact, contrary to a misunderstanding of a quotation highlighted by Merkley and colleagues (2016) and Odic and colleagues (2014) offered several reasons to believe that ANS confidence hysteresis is domain specific. To clarify, for Odic and colleagues, the sense in which confidence hysteresis is "general" is not that a hysteresis-based improvement in one domain (e.g., approximate number) will transfer to another domain (e.g., luminance judgments); rather, Odic and colleagues suggested that hysteresis effects are likely to be observed in any perceptual decision task across a range of domains. Still, further work is needed to understand the mechanisms underlying confidence hysteresis. One plausible contributing mechanism is that confidence hysteresis does affect children's emotional stance toward the task at hand—but in a domain-specific way. Performing ANS discriminations in an Easy-First trial order might, for example, reduce children's feelings of math anxiety and, thereby, lead to improvements in subsequent math performance. This is just one possible mechanism, and we look forward to testing this and other hypotheses.

Merkley and colleagues (2016) bring up a few other concerns. They believe that choosing a subset of items from the standardized Test of Early Mathematics Ability (TEMA-3; Ginsburg & Baroody, 2003), as we did, leads to an unsurprising result. Because these items were previously shown to correlate with ANS precision (Libertus et al., 2013b), they believe it was a given that children in our Easy-First ANS condition should outperform children in our Hard-First ANS condition in symbolic math. However, to us, this criticism misses the logic of our experimental design. Our aim was not to ask whether an ANS manipulation affects all mathematical abilities or the entirety of math abilities probed by standardized tasks like the TEMA. Neither did we wish to present ANS confidence hysteresis as an intervention for engendering lasting improvements in symbolic mathematics. Instead, we aimed to take a previously demonstrated relationship—the relationship between ANS precision and children's performance on a specific set of symbolic math problems—and assess the causal nature of this particular relationship. Our surprising finding is that a 5-min experimental manipulation of ANS precision affected children's performance on a set of problems that do not seem to resemble the manipulated domain itself (i.e., snap judgments about dot arrays vs. the deliberative solving of problems involving Arabic numerals). If what one wants to know is whether manipulations of ANS precision also affect other kinds of math abilities, or affect math abilities on standardized assessments, then our study opens these up as promising avenues for future research.

Finally, Merkley and colleagues (2016) state that "further research is needed to uncover the mechanism underlying the observed correlations between nonsymbolic numerical comparison performance and formal mathematics achievement." We agree wholeheartedly, and we fully embrace all efforts to broaden our methods and approaches as work on these types of questions continues. We look forward to the day when RCT and intervention studies based on in-lab experimental manipulations can be added to this debate. For now, ours is one study in the larger effort, one in which we aimed to discover whether a simple experimental manipulation of the ANS has a detectable effect on children's performance on a subset of math questions. It did. In the end, it will be the sum total of the evidence across studies, using multiple methods, that enables the field to characterize the relationships between the ANS and symbolic math ability. We hope that there will be room for many different approaches, with each one making a small contribution to the much grander enterprise.

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